Is Selection on Firm Productivity a Third Gain from Trade?

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Abstract

Empirical studies provide strong evidence that trade liberalization reallocates resources towards high productivity firms. Theoretical models of international trade have incorporated firm heterogeneity to explain such selection. This paper addresses two questions: Do Selection Effects yield new Gains from Trade distinct from Comparative Advantage and Scale Effects? How does Selection caused by trade compare with domestic policy options? Examining heterogeneous firm models, we find the answers depend not on the production structure rather the demand structure. Linear demand as in Melitz and Ottaviano (2008) generates returns to scale which favor the most productive firms, making Selection in this model a Scale Effect. In contrast, the original model of Melitz (2003) exhibits Selection Effects as a genuinely new Gain from Trade. Selection in a Melitz economy reflects the optimal internalization of trade frictions. We show the Melitz model is efficient, independent of the productivity distribution of firms. The CES demand of Melitz (2003) is necessary for efficiency as is the taste for variety imposed in the model. The results highlight the role of demand in determining when Selection Effects are distinct, optimal and anti-variety.

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1 Introduction

Empirical studies provide strong evidence that trade liberalization induces exit of low productivity firms and increases sales of high productivity firms.[1] To explain these observations, theoretical models have incorporated firm heterogeneity to explain selection of higher productivity firms through international trade. The key insight of this literature is that opening the economy to trade increases competition among firms, leading to a rise in average productivity through reallocation of resources towards more productive firms. Such Selection Effects increase average productivity and result in lower average prices, yielding welfare gains for consumers. This is in contrast to standard monopolistic competition models where trade does not give rise to selection since all firms have the same productivity. Thus, recent models with heterogeneous firms suggest Selection Effects are a new channel for Gains from Trade (GFT).[2] In this paper, we examine the nature of Selection Effects as a source of GFT.

Do Selection Effects provide new welfare gains that cannot be attained without trade? For example, if selection is solely due to tougher competition from larger markets, then selection is actually a scale effect which cannot be attained without trade. Concretely, in the model of Melitz and Ottaviano (2008), increases in domestic scale replicate selection induced by trade. Selection effects are a consequence of larger market size through trade rather than a new GFT.

On the other hand, domestic scale is distinct from Selection Effects in Melitz (2003), implying selection gives potentially new GFT. This raises the question whether welfare gains from selection can be achieved without trade. A thought experiment makes this point clear. Consider a closed economy divided into two identical parts that can trade with each other. In standard comparative advantage and scale effects models, this division does not provide any welfare gains since relative technologies, endowments and scale of the economy stay the same. Therefore comparative advantage and scale effects induced by international trade cannot be replicated in autarky. However, Selection Effects can be replicated in autarky. Suppose firms must pay taxes matching the variable and fixed costs of international trade when exporting to the other part of the country.

[1] See for instance Clerides, Lach, and Tybout (1998); Bernard and Jensen (1999); Aw, Chung, and Roberts (2000); Pavcnik (2002); Trefler (2004); Bernard, Jensen, and Schott (2003); Helpman, Melitz, and Yeaple (2004a); Das, Roberts, and Tybout (2007). For a detailed survey of the literature, the reader is referred to Helpman (2006).

[2] Considering 48 countries exporting to the US in 1980-2000, Feenstra and Kee (2007) estimate that rise in export variety accounts for an average 3.3 per cent rise in productivity and GDP for the exporting country.

Then Selection Effects arise in autarky. However, we show that the closed market equilibrium of Melitz (2003) is efficient so a planner will never choose to emulate selection caused by trade.

Why then does Selection caused by trade generate welfare gains? We find that efficiency in autarky carries over to the open economy with export costs. In the presence of trade costs, the firms a planner would weed out in the open economy are exactly those that would not survive in the open market. Thus market selection is the optimal internalization of trade frictions.

To sum up, Selection is distinct from Scale in Melitz and arises as an optimal internalization of trade frictions. In Melitz and Ottaviano, Selection is a form of Scale implying trade expands production possibilities of the economy. What drives these different outcomes? By ruling out differences on the production side, we find the difference arises from the demand structure of the economy. We show Selection Effects are distinct from Scale Effects under separable preferences. Separable preferences imply that *relative* demand for varieties does not respond to exogenous increases in market size. Therefore, increasing the scale of the economy does not alter profitability of firms, leaving selection unchanged. This result is consistent with work in the context of consumption-leisure choices (e.g. Deaton, 1981). In contrast, the non-separable preferences of Melitz and Ottaviano (2008) imply selection is a form of Scale Effects. In this case, selection occurs because of tougher competition in larger markets. In Melitz, selection arises due to trade frictions and not scale.

As the nature of selection depends so heavily on the demand structure, we then focus on consumer preferences. We show the CES form is unique in ensuring that selection is optimal. Selection Effects of trade may reduce welfare under separable preferences other than those of Melitz (e.g. Benassy-CES of Bilbiie, Ghironi, and Melitz, 2005b).

This paper contributes to the long tradition of examining welfare implications of trade and to recent work focusing on welfare in new trade models. Comparing Selection to Comparative Advantage and Scale Effects, we answer when welfare gains from Selection Effects constitute a third GFT. Recent work has drawn attention to the question of whether Selection Effects would increase empirical estimates for welfare gains. Focusing on standard methods used to measure GFT, Arkolakis, Costinot, and Rodriguez-Clare (2009) show that selection would not change existing empirical measures. Our paper contributes to this literature from a theoretical standpoint.

Selection Effects provide welfare gains by optimally internalizing trade frictions in Melitz.[3] In the presence of trade frictions, Selection solves the informational problem of who gets additional resources from trade. The market selects the same firms as a planner with complete information. Therefore, Selection in Melitz differs from Comparative Advantage and Scale Effects which provide welfare gains by expanding consumption or production possibilities.

The optimality results follow the line of query in Dixit and Stiglitz (1977) and are closely related to Bilbiie, Ghironi, and Melitz (2005a) for symmetric firms. Within the heterogeneous firm literature, Baldwin and Nicoud (2005) and Feenstra and Kee (2006) discuss certain efficiency properties of the Melitz economy. Taking their analysis further, we find market outcomes are socially optimal in a Melitz economy and highlight the special role played by CES demand. These results show the demand side is crucial in determining the nature of welfare gains from selection.[4] In fact, departing from CES preferences, we find Selection Effects are a form of scale and provide real gains through an expansion of production possibilities in Melitz and Ottaviano. Thus our findings echo the observation of Feenstra (2010) who suggests moving beyond the CES case to capture social gains from lower markups. In fact, we find that a variable markup setting increases real welfare through lower markups as well as selection.

The paper is organized as follows. Section 2 recaps the theoretical structure of trade models with firm heterogeneity. In Section 3, we present the relationship between Scale and Selection discussed above and explain how selection on firm productivity generates welfare gains by inducing optimality. Section 4 characterizes the deeper features of demand that relate Selection Effects to Scale and determine the optimality of Selection Effects. Section 5 discusses the results by placing them into the context of the literature and presents conclusions.

[3]Atkeson and Burstein (2005) consider a first-order approximation and numerical exercises for market outcomes in a Melitz economy to show that productivity increases are offset by reductions in variety. We provide an analytical treatment for the market equilibrium and further examine the social optimum to show that selection cannot increase production and consumption possibilities.

[4]Arkolakis, Costinot, and Rodriguez-Clare (2009) and Atkeson and Burstein (2005) make CES assumptions which we show are unique in their welfare implications. Compared to these papers, we only focus on GFT from Selection Effects as our purpose is to examine whether selection constitutes a third source of gain from trade.

2 Trade Models with Heterogeneous Firms

Trade models with heterogeneous firms differ from earlier trade models with product differentiation in two significant ways. First, costs of production are unknown to firms before sunk costs of entry are incurred. Second, firms are asymmetric in their costs of production. This asymmetry of costs induces Selection Effects within an industry. In this section we briefly recap the implications of asymmetric costs for consumers, firms and equilibrium outcomes. Readers familiar with the models may wish simply to refer to a summary of the equilibria in these models provided in Tables 3 and 4 in the Appendix.

2.1 Consumers

A mass L (L^*) of identical consumers in the home (foreign) economy are each endowed with one unit of labor and face a wage rate w normalized to one. Preferences are identical in the home and foreign countries. Each consumer has preferences over differentiated goods $U(M_e, \mathbf{q})$ which induce an inverse demand $D(q(c))$ for each good indexed by c. Preferences are CES[5] in the Melitz economy and quasilinear[6] in Melitz and Ottaviano (MO hereafter). The forms are given respectively in definitions (CES) and (QL).

$$U(M_e, \mathbf{q}) \equiv M_e^{1/\rho} \left(\int (q(c))^{\rho} dG \right)^{1/\rho} \tag{CES}$$

$$U(M_e, \mathbf{q}) \equiv q_0 + \alpha M_e \int q(c) dG - \frac{\gamma}{2} M_e \int q(c)^2 dG - \frac{\eta}{2} \left(M_e \int q(c) dG \right)^2 \tag{QL}$$

The significance of the parameters α, γ, η in MO deserves some mention. An increase in α or a decrease in η shifts demand up for the differentiated good relative to the numeraire good. On the other hand, γ indexes substitution possibilities within the differentiated goods sector with $\gamma = 0$ implying that varieties are perfect substitutes.[7] Given γ, lower average prices and higher number

[5] CES preferences over differentiated goods yield the familiar demand curve ($D(q(c)) = IQ^{-\rho} q(c)^{\rho-1}$) which depends on the elasticity of substitution $1/(1-\rho)$ and the aggregate bundle of goods $Q \equiv U$.

[6] Here the numeraire good is denoted q_0 and α, γ, η are positive parameters. Consumer demand is given by $D(q(c)) = \alpha - \gamma q(c) - \eta Q$ which depends on the aggregate bundle $Q \equiv M_e \int q(c) dG$. Following MO we assume throughout that an interiority condition is met in the market equilibrium so that consumers demand positive amounts of the numeraire good.

[7] These preferences differ significantly from the CES case as marginal utilities are bounded so that consumers need not demand positive amounts of any particular variety. MO note that compared to the CES case, the price elasticity of demand is not solely a function of the elasticity of substitution between varieties (γ in this case).

of competing firms increase the price elasticity of demand. In MO's terminology, "toughness" of competition has an impact on pricing decisions.

2.2 Firms

There is a continuum of firms which may enter the market for differentiated goods, by paying a sunk entry cost of f_e. We denote the mass of all entering firms by M_e. Upon entry, each firm receives a marginal cost draw of c drawn from a distribution G with continuously differentiable pdf g.[8] In MO, G is assumed to be Pareto(k, c_M) with $g(c) \equiv kc^{k-1}/c_M^k, \quad c \in [0, c_M]$. Each firm acts as a monopolist of a distinct variety. Accordingly, we index each differentiated good by c and the quantity and price respectively by $q(c)$ and $p(c)$. After entry, should a firm produce for the domestic market it faces a cost function $\text{TC}(q(c)) \equiv cq(c) + f$ where f denotes the fixed cost of production. Each firm faces an inverse demand of $p(c) = D(q(c))$ and acts as a monopolist, charging a markup over cost denoted by $\mu(c) \equiv p(c) - c$. Post-entry profit of the firm from domestic sales is $\pi(c)$ where $\pi(c) \equiv \max_{q(c)}[p(c) - c]q(c) - f$. When the economy opens to trade, firms incur an iceberg transport cost $\tau^* > 1$ and a fixed cost $f_x \geq 0$ in order to export to other countries. As a result, firms face a cost function $\text{TC}(q_x(c)) \equiv \tau^* cq_x(c) + f_x$ and a demand function $D(q_x(c))$ for sales to the export market (x).

Profit maximization implies that firms produce for the domestic and export markets if they can earn non-negative profits from sales in the domestic and export markets respectively. We denote the cutoff cost level of firms that are indifferent between producing and exiting from the domestic market as c_a in the closed economy and c_d in the open economy. The cutoff cost level for firms indifferent between exporting and not producing for the export market is denoted by c_x. Formally, let $i = a, d, x$ denote autarky and the domestic and export markets in the open home economy respectively. Each c_i is fixed by the Zero Profit Condition (ZPC).

$$\pi_i(c_i) = 0 \qquad \text{For } i = a, d, x \qquad \text{(ZPC)}$$

Since firms with cost draws higher than the cutoff level do not produce, the mass of domestic producers (M_i) supplying to market i is $M_i = M_e G(c_i)$.

[8] Some additional regularity conditions on G are required for existence of a market equilibrium in Melitz.

In summary, each firm faces a two-stage problem: in the second stage it maximizes profits from domestic and export sales given a known cost draw, and in the first stage it decides whether to enter given the expected profits in the second stage. Finally, we maintain the standard free entry condition imposed in monopolistic competition models. Specifically, let $\Pi(c)$ denote the total expected profit from sales in all markets for a firm with cost draw c, then ex ante average Π net of sunk entry costs must be zero.

$$\int \Pi(c) dG = f_e \quad \text{(FE)}$$

The equilibrium outcomes are summarized in Tables 3 and 4 in the Appendix.

3 Selection Effects as a Gain from Trade

In this Section we disentangle Selection from Scale Effects, showing when Selection is a form of previously known Scale effects or a new GFT. Concretely, we show selection in Melitz and Ottaviano (2008) is replicated by scale implying Selection Effects arise due to an increase in market size. In Melitz, we show Selection is a new source of GFT and characterize Selection as an optimal response to trade frictions. Finally, we show that the different roles of Selection in Melitz and MO are due to differences in demand rather than firm costs. This sets the stage for the following section which shows how demand structures influence selection.

3.1 When Selection is a Scale Effect

In order to highlight the nature of selection on firm productivity, we consider trade between countries with a single factor of production and ex ante identical cost distributions. This rules out Heckscher-Ohlin and Ricardian comparative advantage, allowing us to focus on the relationship between Scale Effects and Selection Effects. In MO, Selection is a new form of previously known Scale Effects. In Melitz, Selection is a consequence of trade frictions and not Scale. In this sense, the MO and Melitz frameworks are orthogonal.

Proposition 1. Scale Effects of Trade in Heterogeneous Firm Models

1. Average productivity and markup of the open Melitz and Ottaviano (2008) economy can be replicated in autarky by an appropriate change of scale (L).

2. Average productivity and markup of the closed and open Melitz (2003) economies are independent of scale (L).

Proof. We detail the relationship between scale (L) and average productivity in autarky ($\tilde{c}_a$) and in the open economy ($\tilde{c}_t$). Recall c_a denotes the autarkic domestic cost cutoff and c_d denotes the open economy cost cutoff for domestic sales. In both models, average productivity is fixed by cutoff cost levels of the lowest productivity firms. For Melitz, this is shown algebraically in the Appendix. For MO, the relationships are particularly simple, namely $\tilde{c}_a = \frac{k}{k+1}c_a$ and $\tilde{c}_t = \frac{k}{k+1}c_d$. In both models, the cost cutoffs are fixed by the conditions determining the zero profit level and free entry (ZPC = FE). The ZPC = FE conditions are provided in Table 1.

Table 1: Cutoff Cost Levels and Scale Effects

	Autarky	Open Economy
Melitz	$f_e/G(c_a) = fk(c_a)$	$f_e/G(c_d) = fk(c_d) + nG(c_x(c_d))f_x k(c_x(c_d))/G(c_d)$
MO	$c_a^{k+2} = 2(k+1)(k+2)\gamma c_M^k f_e/L$	$c_d^{k+2} = \frac{1-(\tau^*)^{-k}}{1-(\tau^*)^{-k}(\tau)^{-k}} \cdot 2(k+1)(k+2)\gamma c_M^k f_e/L$

Table 1 shows that ZPC = FE is independent of scale in the Melitz economy so that c_a does not change with L. Therefore the open economy average productivity level cannot be attained by scaling market size in Melitz. In contrast, Table 1 shows that in the MO economy scale does affect cost cutoffs and thus average markup and productivity.

We now show that by appropriate scaling of a closed MO economy of size L to size sL, the autarkic productivity of the scaled economy, $\tilde{c}_s$ can replicate the open economy productivity $\tilde{c}_t$. Referring to Table 1, $\tilde{c}_s = \tilde{c}_t$ when

$$s = [1 - (\tau^*)^{-k}(\tau)^{-k}]/[1 - (\tau^*)^{-k}] \qquad \text{(MO Scale)}$$

Thus scaling up resources in a closed MO economy from L to sL yields the open economy average productivity level in autarky. As expected when $\tau = \tau^* = 1$, the scaling factor $s = 2$. □

The first consequence of Proposition 1 is Selection Effects cannot be a new source of GFT in the MO economy since they are really Scale Effects. The second consequence of Proposition 1 is that in the Melitz economy, Selection Effects are unrelated to scale. These results are summarized below.

Corollary. Interdependence of Scale and Selection

1. Selection Effects are a form of Scale Effects in MO. In particular they can be replicated by scaling autarkic resources.

2. Selection Effects are distinct from Scale Effects in Melitz (2003).[9]

[9] This Corollary explains the observation of Baldwin and Forslid (2006) that reduction in trade costs can have an "anti-variety effect" in a Melitz economy. Indeed, Arkolakis, Demidova, Klenow, and Rodriguez-Clare (2008) provide

3.2 How Selection Generates GFT

We now examine how international trade generates welfare gains through Selection. As shown above, in a MO economy Selection Effects are a form of Scale Effects. Therefore, international trade provides welfare gains from selection by expanding production possibilities of a MO economy. In a Melitz economy, Selection Effects are independent of scale and the mechanism for welfare gains differs from traditional GFT. Consequently, we provide a detailed exposition of how Selection generates welfare gains in the Melitz model.

We first show there is nothing special about international trade in inducing selection. In fact, Selection Effects can be replicated in autarky by using taxes similar to trade costs. To see this, consider a closed Melitz economy divided into two identical parts that can trade with each other. Suppose firms must pay taxes (τ, f_x) when exporting to the other part of the country. Then the firm problem is exactly as in an open Melitz economy, though the scale of this economy is lower than the open economy. As shown in Proposition 1, scale does not matter for cost cutoffs c_d and consequently resource allocation across firms in the economy is exactly as in the open Melitz economy. Selection Effects of the open economy are replicated.

Since Selection is welfare improving in the open economy and tax policy can replicate Selection induced by trade, can Selection improve welfare in autarky? The answer is no; we find that the autarkic market equilibrium is efficient. This implies that Selection associated with trade is undesirable in autarky. It reduces entry and generates too little variety. However, the open economy market equilibrium is efficient as well. How can Selection in the open economy be inefficient in autarky? As discussed below, the difference occurs because GFT from Selection is an optimal internalization of trade frictions. Conditional on trade costs, the market induces optimal Selection Effects in a Melitz economy. We state the efficiency results as Propositions 2 and 3.

Proposition 2. *Every market equilibrium of a closed Melitz economy is socially optimal.*

Proposition 3. *Every market equilibrium of an open Melitz economy is socially optimal.*

Proofs. See Appendix. □

evidence of only small gains from variety in Costa Rica. These findings are a consequence of the relationship between Selection and Scale. In a Melitz economy, Selection Effects are distinct from Scale Effects. Thus, there is no reason for them to move together. In fact, higher fixed exporting costs intensify selection but reduce entry due to lower expected profits. On the other hand, in a MO economy, Selection Effects are a form of Scale Effects so trade frictions affect variety and selection in the same direction. There is no anti-variety effect as Selection is also a form of Scale.

Although the proofs of Propositions 2 and 3 are involved, the reasoning is similar to that in homogeneous firm monopolistic competition models. From that literature we know that CES preferences yield "synchronization of markups" across firms (Bilbiie, Ghironi, and Melitz 2005b). The synchronized markups and free entry imply that changes in consumer surplus and profits exactly balance each other, leading to an efficient market equilibrium (Grossman and Helpman 1993, see also Baldwin and Nicoud 2005). Propositions 2 and 3 show these properties are preserved in a heterogeneous firm setup with sunk costs.

Efficiency of the market equilibrium in a Melitz economy is tied to the CES assumption. CES preferences and free entry imply market allocations are socially optimal. The market makes an "*optimal departure from marginal cost pricing*" and replicates the social optimum. Firms charge markups but free entry implies the proceeds from markups exactly finance the fixed and sunk costs of production. Isoelasticity of preferences ensures constant markups across firms. Thus, prices are proportionate to costs and the market quantities are proportionate to the quantities produced under marginal cost pricing. This result is consistent with the insight of Baumol and Bradford (1970) that in the presence of fixed costs, Pareto optimal utilization of resources requires proportionate reductions in all quantities.

In conclusion, both closed and open market outcomes are efficient. Differences in Selection occur because variable profits in the open economy reflect additional trade frictions. The market internalizes these frictions and eliminates firms with low profitability. We detail this process next.

3.3 Selection as an Optimal Response to Trade Frictions

Why does Selection yield welfare gains in an open economy? We address this question in the original setting of Melitz (2003). Here Selection arises as an optimal response to trade frictions, yielding positive welfare gains.

To highlight this clearly, consider a Melitz economy which stays closed but has twice its original scale $2L$. This controls for Scale Effects from two identically sized economies trading with each other. Having controlled for Scale Effects, divide the scaled closed economy into two parts of size L. Suppose a planner implements a tax policy where firms must pay variable and fixed costs (τ, f_x) to trade with each other. In such a scaled closed economy, the firm problem is identical to an open Melitz economy, and open economy firm outcomes are replicated. Consequently, controlling for scale, Selection can be generated in autarky by imposing frictions such as taxes (τ, f_x). Put differently, international trade by itself does not induce selection. Selection is a consequence of frictions associated with trade and can be generated even in autarky.

From Proposition 2, a no tax policy is optimal in a scaled closed economy since the market is efficient. This implies welfare in a scaled closed economy (with no selection) is higher than welfare in an open Melitz economy with selection. However, Proposition 3 implies that if an increase in scale can only be attained at a cost of exogenous frictions (τ, f_x), Selection Effects are optimal. We summarize this result as Proposition 4.

Proposition 4. *In a Melitz economy, Selection Effects induced by international trade can be generated in autarky but are not socially optimal. If new markets can only be accessed by paying* (τ, f_x) *then these Selection Effects are optimal.*

Proposition 4 shows Selection Effects in a Melitz economy are an internalization of trade frictions and do not reflect "real" gains on the production or consumption sides. However, Selection Effects are an optimal internalization of trade frictions and hence yield positive GFT in the presence of trade frictions. Thus, Selection Effects in Melitz perform the function of allocating additional resources optimally without any informational requirements. We now contrast these results with selection in a MO economy.

3.4 Why Selection Differs in Melitz and MO

There are three key differences between the Melitz and MO economies:

1. Fixed costs of exporting (f_x) in Melitz,
2. The presence of a homogeneous good in MO,
3. Demand for differentiated goods.

We harmonize MO to Melitz along the lines of fixed costs and homogeneous goods. As detailed below, we find that Selection is still a Scale Effect. Therefore the independence of Selection from Scale in Melitz and not in MO can only be due to demand for differentiated goods.[10]

[10] We also remark the assumption of a single sector in Melitz is not driving the novelty. Bernard, Redding, and Schott (2007) use a two-sector Cobb-Douglas Melitz framework and derive cost cutoff levels for each sector. Examination

Adding Fixed Costs of Exporting to MO. In the Melitz economy firms incur a fixed cost to enter export markets while in the MO economy these fixed costs of exporting are assumed to be zero. Selection Effects vanish in the Melitz model when $f_x = 0$ so we might expect that once f_x is incorporated into the MO model, Selection Effects may be independent of scale. Accordingly, we consider a case where all assumptions of the MO model are retained but now firms have to incur a fixed cost for exporting ($f_x > 0$). It turns out that after introducing such fixed costs in the MO model, Proposition 1 still holds so Selection Effects are still a form of Scale Effects. In MO, the presence of homogeneous goods implies the supply of labor to the differentiated goods sector is elastic. So it is not surprising that fixed exporting costs f_x play a role similar to iceberg transport costs τ.

Removing Homogeneous Goods from MO. Unlike Melitz, MO consider preferences defined over a homogeneous good in addition to the differentiated good. We relax this assumption to examine whether the presence of a homogeneous good prevents independence of Selection Effects from Scale Effects in MO. For illustrative purposes, we consider only symmetric resource endowments and trade costs. After removing homogeneous goods from MO, we find preferences continue to be non-separable so Selection Effects remain Scale Effects.

In conclusion, the independence of Selection as a GFT is not driven by the supply side of the model. Rather, different demand schedules induce monopolistically competitive firms to respond differently following a change in market size. In the next Section, we further explore the role of preferences in generating independence and efficiency.

4 Demand: Selection, Scale and Efficiency

In a Melitz economy, Selection Effects are distinct from scale and are optimal in the presence of trade frictions. As pointed out in the last Section, these properties are a consequence of demand schedules. In this Section, we detail the role of demand in distinguishing selection from scale and inducing efficient selection.

First, we show firm selection is tied to how scale shifts demand for varieties. We then show that separable preferences imply demand for varieties is independent of scale. Therefore separable

shows these cutoffs are independent of scale implying that Melitz (2003) may be extended to a multisector framework which preserves Proposition 1.

preferences make selection independent of scale, as in Melitz (2003).[11] In contrast, scale shifts demand in MO, inducing selection.

Second, we show that isoelasticity is necessary to ensure market efficiency. Trade leads to optimal Selection only when preferences have the CES form. However, isoelasticity alone does not produce optimal Selection. Incorporating a "taste for variety" along the lines of Benassy (1996) will cause the market to over or under select firms.[12]

The results in this section emphasize how properties of consumer demand impact Selection. We summarize the results in the context of the wider literature in the next section.

4.1 Demand Shifts and Selection

Firm selection is clearly tied to the demand for varieties $p(q(c))$. For instance, if a change in scale L increases the price the least productive firm c_a receives ($\partial p(q(c_a))/\partial L > 0$) then c_a will make positive variable profits and less productive firms will enter ($\partial c_a/\partial L > 0$). Similarly, shifts in the entire demand curve for varieties ($\partial p(q(c))/\partial L$) change c_a through the effect on firm revenues. To see this explicitly, recall the free entry condition

$$\int_0^{c_a} \{[p(q(c)) - c]q(c) - f\} \, dG = f_e$$

Differentiating with respect to scale L and appealing to the envelope theorem (since $q(c)$ is the *optimal* quantity of a firm) we know

$$\int_0^{c_a} [\partial p(q(c))/\partial L + \partial p(q(c))/\partial c_a \cdot \partial c_a/\partial L] q(c) dG +$$
$$[\partial c_a/\partial L] \cdot \{[p(q(c_a)) - c_a]q(c_a) - f\} \cdot G'(c_a) = 0$$

By definition, a firm with cost draw c_a makes zero profits $[p(q(c_a)) - c_a]q(c_a) - f = 0$ implying

$$\int \partial p(q(c))/\partial L \cdot q(c) dG = -\partial c_a/\partial L \cdot \int \partial p(q(c))/\partial c_a \cdot q(c) dG$$

[11] In particular, independence from scale is not due to the assumption of isoelasticity.

[12] We illustrate the unique welfare properties by generalizing Melitz to Benassy preferences. Alternative generalizations could include an additive numeraire (Bilbiie, Ghironi, and Melitz 2005a), terms of trade effects (Demidova and Rodriguez-Clare 2007), asymmetric countries or trade costs etc.

As monopolistic competition implies additional competitors lower prices ($\partial p(q(c))/\partial c_a < 0$), it follows that $\partial c_a/\partial L$ and $\int \partial p(q(c))/\partial L \cdot q(c)dG$ have the same signs. The term $\int \partial p(q(c))/\partial L \cdot q(c)dG$ is the effect of scale on revenue at current quantities. For example, if scale shifts demand up ($\partial p(q(c))/\partial L > 0$) then higher revenues allow higher cost firms to produce. This result relates the effect of scale on selection to demand shifts, summarized in Proposition 5.

Proposition 5. *Selection is independent of scale if and only if demand shifts from scale are revenue neutral at current quantities. Formally,*

$$\partial c_a/\partial L = 0 \quad \textit{iff} \quad \int_0^{c_a} \partial p(q(c))/\partial L \cdot q(c)dG = 0$$

This proposition helps explain why selection is distinct from scale in Melitz (2003) but not in MO. Consider the general class of "Melitz-type" preferences of the form given by Equation (1):

$$U(M_e, c_a, q) \equiv \nu(M_e, c_a) \int_0^{c_a} u(q(c))dG \tag{1}$$

Here u denotes utility from the bundle of differentiated goods while the function ν denotes utility from mass of variety.[13] Melitz-type preferences of Equation (1) are weakly separable in the bundle of differentiated goods $q(c)$ and mass of entrants M_e. As in two-stage budgeting problems, a change in market size only changes the shares allocated to mass of variety and the unit bundle, but not substitution within the bundle. Formally, $\partial p(q(c))/\partial L = 0$, so Proposition 5 shows that selection is unresponsive to L. Of course, Equation (1) includes Melitz as a special case. We state this formally in Proposition 6.

Proposition 6. *Let preferences be separable as in Equation (1). Then scale does not matter for the cost cutoff level so Selection Effects are distinct from Scale Effects.*

Proof. See Appendix. □

In contrast, separability breaks down in MO; an increase in market size changes both quantities and variety. Relative demand for varieties shifts with scale. To see that these scale-induced demand shifts must affect selection, we appeal again to Proposition 5. Suppose $\partial c_a/\partial L = 0$ so prices for the c_a firm must also be constant ($\partial p(q(c_a))/\partial L = 0$). At the same time, we know demand is linear and pivoting to a new line through the point $(q(c_a), p(q(c_a)))$. Since $\partial p(q(c))/\partial L$

[13]We assume ν is positive and continuously differentiable and u satisfies usual regularity conditions which guarantee that each monopolist will have a unique optimal quantity in a market equilibrium. Note that the functions u and ν can be defined to yield variable own-price elasticities that need not be equal across firms or to the elasticity of substitution.

represents a line pivoting through a point, either demand is shifting up ($\partial p(q(c))/\partial L > 0$) or down ($\partial p(q(c))/\partial L < 0$), which violates revenue neutrality. We must conclude $\partial c_a/\partial L \neq 0$, so scale affects selection in MO.

4.2 Optimal Selection: Isoelasticity and Taste for Variety

The welfare-enhancing effects of international trade and their efficiency properties have figured prominently in the literature. Within heterogeneous firm models, Feenstra and Kee (2006) show that quantities and cost cutoffs are optimal in a Melitz economy when industry prices are held constant. We have taken their analysis further by showing the market equilibrium of an autarkic and open Melitz economy is efficient and that Selection Effects are an optimal internalization of trade frictions. The significance of CES preferences of Melitz cannot be understated. Departing from CES preferences, the market equilibrium and the adjustment to trade frictions are no longer socially optimal. This holds even for general Melitz-type preferences of Equation (1) as stated in Proposition 7.

Proposition 7. *Consider an economy with preferences of Equation (1). A necessary condition for the market equilibrium to be socially optimal is that u is CES.*

Proof. See Appendix. □

Proposition 6 implies if utility from the unit bundle is not CES, i.e. $u(q(c)) \neq q(c)^\rho$, then Selection Effects cannot be optimal. Except in the CES case, trade does not lead to an optimal internalization of trade frictions. When the economy opens to trade, Selection Effects arise in the market equilibrium but there is no guarantee that they are desirable. Depending on the utility specification of u and ν, Selection Effects may not even be welfare-improving. In such cases, the market selects too much or too little and variety gets too low or too high.

We now detail a specific example of inefficient Selection arising from "taste for variety". Following Bilbiie, Ghironi, and Melitz (2005b), we specialize the separable preferences of Equation (1) to CES-Benassy preferences which disentangle "taste for variety" from market power (Benassy, 1996). The specialization assumes isoelasticity and defines $\nu(M_e, c_a) \equiv [M_e G(c_d)]^{\rho(\nu_B+1)-1} M_e$ which results in preferences similar to that of Melitz with the addition of a Benassy parameter ν_B. The Benassy parameter ν_B captures taste for variety, while market power is given by the markup to cost ratio $(1-\rho)/\rho$. In the special case of Melitz preferences, taste for variety exactly equals the market power of producers ($\nu_B = (1-\rho)/\rho$), in which case $\nu(M_e, c_a) = M_e$.

Once taste for variety is incorporated, isoelasticity is no longer sufficient to produce optimal selection. Proposition 8 shows Selection is optimal iff taste for variety equals market power.

Proposition 8. *Consider an economy with preferences of Equation (1) and Benassy ν. The market equilibrium is socially optimal only if u is CES and taste for variety equals market power.*

Proof. See Appendix. □

Proposition 8 is the classic Chamberlinian "efficiency versus diversity" problem re-interpreted in the presence of selection (Chamberlin 1950). It highlights whether the market provides the optimal levels of quantity versus variety. With the exception of $\nu_B = (1-\rho)/\rho$, the market fails to optimally select the cost cutoff for firms and induces suboptimal levels of quantity versus variety. With the results for Selection in hand, the next section places Selection in the context of the wider literature on gains from trade.

5 Discussion and Conclusion

In this Section we summarize our results on Selection and place them in the context of other Gains from Trade, in particular Comparative Advantage and Scale Effects (see Table 2).[14]

5.1 GFT: Comparative Advantage and Scale Effects

As shown in Section 3, Selection Effects in Melitz and Ottaviano (2008) are a form of Scale Effects while Selection is distinct from Scale in Melitz (2003). These results are presented in the bottom of Column (a) of Table 2. In what follows, we relate key results for Selection in each model with other sources of GFT.

Comparative Advantage. In standard comparative advantage models, aggregate gains from trade on the production and consumption sides result in a Pareto improvement in the economy once distributional issues are addressed.[15] In the absence of trade, resources can be reallocated

[14]See Facchini and Willmann (1999); Helpman and Krugman (1985, Chap. 9) for a survey of Gains from Trade in constant returns to scale and increasing returns to scale models of trade respectively. Also see Feenstra (2006) for a survey of the empirical evidence on gains from trade.

[15]Kemp and Wan (1972); Dixit and Norman (1988). For a detailed survey regarding distributional policies see Facchini and Willmann (1999).

Table 2: Comparison of Selection Effects with Comparative Advantage and Scale Effects

	GFT Source	(a) Distinct From Other GFT	(b) GFT Mechanism	(c) Pareto Efficient
1.	*Comparative Adv*			
	Ricardo/HO	Yes	Expands Consumption Set	Yes First Welfare Thm
2.	*Scale Effects*			
	Krugman (1980)	Yes	Expands Consumption Set	Yes CES Preferences
3.	***Selection Effects***			
a.	**Melitz (2003)**	Yes **Separable Prefs**	**Optimally Internalize Trade Frictions**	**Yes†** **CES Preferences**
b.	**MO (2008)**	**No Scale Effect**	**Expands Production Set**	**No Variable Markups**

† Efficiency is unique to both CES preferences and the specific "taste for variety" considered by Melitz.

within countries but there is no change in consumption possibilities. Thus trade based on Comparative Advantage expands the consumption set, yielding welfare gains that are unattainable without trade. Comparative Advantage results in a trading equilibrium that is Pareto superior to the autarky equilibrium. In fact, the trading equilibrium is efficient implying no role for trade and industrial policy to improve world welfare. Other sources of GFT are absent in standard Ricardian and Heckscher-Ohlin models implying Comparative Advantage is a new GFT. We summarize these facts in Row (1) of Table 2.

Scale Effects. In standard models with Scale Effects, international trade yields welfare gains that are unattainable without trade (e.g. Helpman and Krugman, 1985). In the absence of a domestic growth mechanism, an economy cannot expand its scale and hence its production or consumption set without engaging in trade. Efficiency of market equilibria have been studied at length in symmetric firm models with Scale Effects.[16] Recently, Bilbiie, Ghironi, and Melitz (2005b) show the market equilibrium is socially optimal if and only if preferences are CES. We generalize their result to models with heterogeneous firms.

Selection Effects. Like Comparative Advantage and Scale Effects, Selection Effects in Melitz (2003) are distinct from other sources of GFT and yield a Pareto efficient equilibrium. In autarky, Selection Effects in Melitz can be emulated through domestic policy, but only at a welfare cost.

[16] Dixit and Stiglitz (1977); Spence (1976); Bilbiie, Ghironi, and Melitz (2005a); Behrens and Murata (2006).

Thus, they do not provide "real" gains in production or consumption possibilities. However, Selection Effects in an open Melitz economy provide welfare gains *to the extent that* they are an optimal internalization of trade frictions. This implies a laissez-faire trade and industrial policy is jointly optimal for the world economy.[17]

However, efficiency holds if and only if preferences are CES. Surprisingly, efficiency is unrelated to the productivity distribution of firms. Allowing for the Benassy-CES preferences of Bilbiie, Ghironi, and Melitz (2005b), we show the market may select too many or too few firms. We summarize these findings in Row (3a) of Table 2.

In contrast, Selection Effects in a Melitz-Ottaviano economy are a form of Scale Effects which yield real gains on the production side as summarized in Row (3b) of Table 2. In MO, the GFT from Selection is not fully realized by the market. The reader may verify that linear demand provides firms with excess market power which drives up profits and eliminates too few firms. This opens the door to industrial policy both in autarky and under trade.[18]

5.2 Conclusion

We have examined the nature of Gains from Trade through Selection Effects in benchmark models with heterogeneous firms. We find that the nature of Selection Effects depends heavily on the structure of consumer demand. With linear demand of Melitz and Ottaviano (2008), Selection Effects are a form of Scale Effects. In the CES model of Melitz, Selection Effects are a new source of GFT, independent of Scale Effects. CES preferences are not unique in their ability to induce this independence of selection and scale. For a general class of separable preferences, demand and entry changes offset each other leading to independence of selection and scaling. So for a broad class of preferences, Selection Effects are a new source of GFT. Unlike traditional sources of GFT, Selection Effects in the Melitz economy are an internalization of trade frictions. The market acts as an information aggregator by allocating additional resources optimally. However, this optimality property is unique to CES preferences, and breaks down even if "taste for variety" is considered. So in principle, Selection Effects need not be optimal or even welfare-improving.

[17]However, terms of trade externalities may exist and lead to a breakdown of laissez-faire policies. Demidova and Rodriguez-Clare (2007) incorporate terms of trade considerations and provide domestic policies to obtain the first-best allocation in an open Melitz economy with Pareto cost draws.

[18]In similar work on optimal policy, Chor (2006) shows domestic tax and subsidy schemes increase welfare in the open economy of Helpman, Melitz, and Yeaple (2004b).

We conclude Selection Effects differ from traditional sources of Gains from Trade. The demand structure and hence, the channel through which Selection operates is crucial in determining the nature of Selection Effects as a third Gain from Trade.

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A Appendix: Proofs

A.1 A Folk Theorem

In this context we need to define the Social Planner's policy space. Provided M_e and $q(c)$, and assuming without loss of generality that all of $q(c)$ is consumed, all allocations are determined. The only question remaining is what class of $q(c)$ the SP is allowed to choose from. A sufficiently rich class for our purposes are $q(c)$ which are positive and continuously differentiable on some closed interval and zero otherwise. This follows from the basic principle that a SP will utilize low cost firms before higher cost firms. Formally, we restrict $q \in \mathcal{Q} \equiv \bigcup_{c_d \in \mathbb{R}} \mathcal{Q}_{[0,c_d]}$ where

$$\mathcal{Q}_{[0,c_d]} \equiv \{q \in \mathcal{C}^1, > 0 \text{ on } [0, c_d] \text{ and } 0 \text{ otherwise}\}$$

so $\mathcal{Q}_{[0,c_d]}$ denotes all smooth, strictly positive quantity allocations on $[0, c_d]$. For brevity we will use the shorthand $G(x) \equiv \int_0^x g(c)dc$ and $R(x) \equiv \int_0^x c^{\frac{\rho}{\rho-1}} g(c)dc$ throughout the proofs. Finally, we maintain Melitz's assumptions which imply a unique market equilibrium in each economy.

Proposition. *Every market equilibrium of a closed Melitz economy is socially optimal.*

Proof. Assume a market equilibrium exists, which guarantees that $R(c)$ is finite for all c. We first remark that in both the market equilibrium and SP problem, $L/M_e - f_e - fG(c_d) = 0$ implies utility of zero so in both cases we must have $L/M_e - f_e - fG(c_d) > 0$. The SP problem is equivalent to

$$\max_{M_e, c_d, q \in \mathcal{Q}_{[0,c_d]}} M_e \int_0^{c_d} q(c)^\rho g(c)dc \text{ subject to } f_e + fG(c_d) + \int_0^{c_d} cq(c)g(c)dc = L/M_e \quad \text{(SP)}$$

We will exhibit a globally optimal $q^*(c)$ for each fixed (M_e, c_d) pair, reducing the SP problem to a choice of M_e and c_d. We then solve for M_e as a function of c_d and finally solve for c_d.

Finding $q^*(c)$ for M_e, c_d fixed. For convenience, define the functionals $V(q), H(q)$ by

$$V(q) \equiv \int_0^{c_d} v(c, q(c))dc \qquad H(q) \equiv \int_0^{c_d} h(c, q(c))dc$$

where $h(c,x) \equiv xcg(c)$ and $v(c,x) \equiv x^\rho g(c)$. One may show that $V(q) - \lambda H(q)$ is strictly concave $\forall \lambda$.[19] Now for each fixed pair (M_e, c_d), consider the problem of finding q^* in $\mathcal{Q}_{[0,c_d]}$ given by

$$\max_{M_e, c_d, q \in \mathcal{Q}_{[0,c_d]}} V(q) \text{ subject to } H(q) = L/M_e - f_e - fG(c_d)$$

Following Troutman Troutman (1996), if some q^* maximizes $V(q) - \lambda H(q)$ on $\mathcal{Q}_{[0,c_d]}$ for some λ then it is a solution to the above problem provided the constraint is met by q^*. Again following Troutman, for any λ a sufficient condition for some $q^* \in \mathcal{Q}_{[0,c_d]}$ to be a global maximum on $\mathcal{Q}_{[0,c_d]}$ is

$$D_2 v(c, q^*(c)) \quad = \quad \lambda D_2 h(c, q^*(c)) \tag{2}$$

This follows because (2) implies for any such q^*, $\forall \xi$ s.t. $q^* + \xi \in \mathcal{Q}_{[0,c_d]}$ we have $\delta V(q^*; \xi) = \lambda \delta H(q^*; \xi)$ (where δ denotes the Gateaux derivative in the direction of ξ) and q^* is a global max since $V(q) - \lambda H(q)$ is strictly concave. The condition (2) is nothing but $\rho q^*(c)^{\rho-1} g(c) = \lambda c g(c)$ which is equivalent[20] to $q^*(c) = (\frac{\lambda c}{\rho})^{\frac{1}{\rho-1}}$. From above, this q^* serves as a solution to $\max V(q)$ provided that $H(q^*) = L/M_e - f_e - fG(c_d)$. This will be satisfied by appropriate choice of λ since for fixed λ we have

$$H(q^*) \quad = \quad \int_0^{c_d} (\frac{\lambda c}{\rho})^{\frac{1}{\rho-1}} c g(c) dc = (\frac{\lambda}{\rho})^{\frac{1}{\rho-1}} R(c_d)$$

so that choosing λ^* as $\lambda^* \equiv \rho[\frac{L/M_e - f_e - fG(c_d)}{R(c_d)}]^{\rho-1}$ will make q^* a solution provided $L/M_e - f_e - fG(c_d) > 0$. In summary, for each fixed (M_e, c_d) a globally optimal q^* satisfying the resource constraint given by

$$q^*(c) \quad = \quad \frac{L/M_e - f_e - fG(c_d)}{R(c_d)} c^{\frac{1}{\rho-1}}$$

which must be > 0 since $L/M_e - f_e - fG(c_d)$ must be > 0 as discussed at the beginning of the proof.

Finding M_e for c_d fixed. We may therefore considering maximizing $W(M_e, c_d)$ where, we have

$$W(M_e, c_d) \equiv M_e \int_0^{c_d} q^*(c)^\rho g(c) dc = M_e [L/M_e - f_e - fG(c_d)]^\rho R(c_d)^{1-\rho}$$

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Since h is linear in x, H is linear and since v is strictly concave in x (using $\rho < 1$) so is V, and therefore $V(q) - \lambda H(q)$ is strictly concave $\forall \lambda$.

20 By abuse of notation we allow q^* to be ∞ at $c = 0$ since reformulation of the problem omitting this single point makes no difference to allocations or utility which are all eventually integrated.

Direct investigation yields

$$\begin{aligned} D_1 W(M_e, c_d) &= [L/M_e - f_e - fG(c_d)]^{\rho-1}\{(1-\rho)L/M_e - f_e - fG(c_d)\}R(c_d)^{1-\rho} \\ D_{11} W(M_e, c_d) &= -\rho(1-\rho)\frac{L^2}{M_e^3}[L/M_e - f_e - fG(c_d)]^{\rho-2}R(c_d)^{1-\rho} \end{aligned}$$

So we conclude that W is strictly concave in M_e and that for each c_d, the globally optimal M_e is given by $M_e(c_d) = (1-\rho)L/(f_e + fG(c_d))$.

Finding c_d. Finally, we have maximal welfare for each fixed c_d, explicitly $Z(c_d)$ where

$$Z(c_d) \equiv \rho^\rho(1-\rho)^{1-\rho}LR(c_d)^{1-\rho}[f_e + fG(c_d)]^{\rho-1}$$

We may easily rule out $c_d = 0$ as an optimum since this yields zero utility. Since $Z(c_d) > 0$ for $c_d > 0$ we may find a maximum for $Z(c_d)$ by maximizing $\ln Z(c_d)$. Some algebra shows that any maximum of $\ln Z(c_d)$ is a maximum of $B(c_d)$ where $B(c_d) \equiv \ln R(c_d) - \ln[f_e + fG(c_d)]$ so we need maximize only $B(c_d)$ which is easily seen to be continuously differentiable on $(0, \infty)$. Now direct inspection shows that

$$B'(c_d) = \{R(c_d)[f_e/f + G(c_d)]\}^{-1}\{R'(c_d)[f_e/f + G(c_d)] - R(c_d)g(c_d)\}$$

Our strategy to find an optimal c_d will be as follows: We will show that $\lim_{c_d \longrightarrow 0} B'(c_d) > 0$ and $\lim_{c_d \longrightarrow \infty} B'(c_d) < 0$ so that for some interval $[a, b]$ with $a > 0$ that $[a, b]$ contains a critical point and we have $\forall \epsilon > 0$ that

$$\sup_{x\in[\epsilon,a]} B(x), \sup_{x\in[b,\infty)} B(x) < \sup_{x\in[a,b]} B(x)$$

We may therefore conclude that B attains a maximum on $[a, b]$ compact since it is continuous. Then since B is continuously differentiable, its maximum must occur at a critical point in $[a, b]$ or at a or b. Since maxima at a or b are ruled out by the above inequalities, we conclude that at least one critical point of B in $[a, b]$ is a global maximum. Finally, by showing that B has a unique critical point, we conclude that B takes on a unique global maximum on $(0, \infty)$.

We now show $\lim_{c_d \longrightarrow 0} B'(c_d) > 0$. Inspection of the expression for B' shows for the first expression it is sufficient to show that for sufficiently small c_d we have

$$\frac{R'(c_d)}{g(c_d)}[f_e/f + G(c_d)] > R(c_d) \tag{3}$$

Since $R(c_d)$ is bounded, it is sufficient that to show that $\lim_{c_d \longrightarrow 0} \frac{R'(c_d)}{g(c_d)} = \infty$, which follows from $\frac{R'(c_d)}{g(c_d)} = c_d^{\frac{\rho}{\rho-1}}$ so we conclude (3).

We now show $\lim_{c_d \longrightarrow \infty} B'(c_d) < 0$. As above it is sufficient to show that $\lim_{c_d \longrightarrow \infty} \frac{f_e/f + G(c_d)}{\int_0^{c_d} (\frac{c_d}{c})^{\frac{\rho}{1-\rho}} g(c)dc} < 1$. Considering the denominator, since $(\frac{c_d}{c})^{\frac{\rho}{1-\rho}} \geq 1$ and g has support on $(0, \infty)$ we conclude that $\frac{G(c_d)}{\int_0^{c_d} (\frac{c_d}{c})^{\frac{\rho}{1-\rho}} g(c)dc} < 1 \quad \forall c_d > 0$. It is therefore sufficient to note that $\lim_{c_d \longrightarrow \infty} \int_0^{c_d} (\frac{c_d}{c})^{\frac{\rho}{1-\rho}} g(c)dc = \infty$ and we conclude $\lim_{c_d \longrightarrow \infty} B'(c_d) < 0$.

All that remains is to show that B has at most one critical point. As above $B'(c_d) = 0$ iff

$$f_e/f \quad = \quad \int_0^{c_d} (\frac{c_d}{c})^{\frac{\rho}{1-\rho}} g(c)dc - G(c_d) \tag{4}$$

and direct inspection shows that the RHS of Equation (4) has a strictly positive derivative in c_d so B has as most one critical point.

Finally, we leave it to the reader to verify that the implicit equation determining c_d is the same as in the market equilibrium, which also determines M_e and q^* to exactly coincide with the market equilibrium.

A.2 Converse of the Folk Theorem

We now consider general consumer preferences of the form given by Equation (5).

$$U(M_e, c_d, q) \equiv \nu(M_e, c_d) \int_0^{c_d} u(q(c))g(c)dc \tag{5}$$

We assume the following regularity conditions on u which guarantee that each monopolist will have a unique optimal quantity in a market equilibrium.

Definition. (Regular Preferences) u satisfies the following conditions:

1. u is twice continuously differentiable with $u' > 0$ and $u'' < 0$ and satisfies the inada conditions.
2. u is s.t. each monopolist's FOC uniquely determines his optimal quantity supplied.[21]
3. ν is positive and C^1.

Proposition 5. *Consider a Melitz economy with preferences as in (1). The market equilibrium is socially optimal only if u is CES.*

Proof. Assume an equilibrium exists which is socially optimal with M_e and c_d fixed by that equilibrium. Also let $q^*(c)$ denote equilibrium quantities. If the equilibrium is efficient for these fixed M_e and c_d, the quantities $q_p(c)$ a planner would choose must be optimal. For convenience, define the functional $H(q)$ as in the above proof and let $U^*(q) \equiv U(M_e, c_d, q)$ be as in Equation (5). By Theorems 5.11 and 5.15 of Troutman, a necessary condition for q_p to be optimal is that either $\delta H(q_p; \xi) = 0 \; \forall \xi \in \mathcal{C}^1[0, c_d]$ or $\exists \lambda$ s.t. $\delta U^*(q_p) = \lambda \delta H(q_p; \xi) = 0 \; \forall \xi \in \mathcal{C}^1[0, c_d]$. We will rule out the first and exploit an implication of the second.

Case 1: $\delta H(q_p; \xi) = 0 \; \forall \xi \in \mathcal{C}^1[0, c_d]$. $\forall \xi$ we have that

$$\delta H(q_p; \xi) = \int_0^{c_d} \xi(c) c g(c) dc = 0$$

which implies $cg(c)$ is identically zero on $[0, c_d]$ by Corollary 4.3 of Troutman which is clearly not optimal.

Case 2: $\delta U^*(q_p) = \lambda \delta H(q_p; \xi) \; \forall \xi \in \mathcal{C}^1[0, c_d]$. For any fixed M_e and c_d and $\forall \xi$ we have that

$$\nu(M_e, c_d) \int_0^{c_d} \xi(c) u'(q_p(c)) g(c) dc = \lambda M_e \int_0^{c_d} \xi(c) c g(c) dc$$

[21] Sufficient conditions for this are $2u'' + u'''q < 0$ or that u is the integral of a strictly decreasing and concave function.

so for $\lambda' \equiv \frac{\lambda M_e}{\nu(M_e,c_d)}$ we have $\int_0^{c_d}[u'(q_p(c)) - \lambda' c]g(c)\xi(c)dc = 0$ and since g is $\mathcal{C}^1$ and strictly positive, again by Corollary 4.3 of Troutman we conclude

$$u'(q_p(c)) = \lambda' c \tag{6}$$

Using similar reasoning, the solution to the consumer's problem in the market equilibrium must necessarily satisfy $u'(q^*(c)) = \mu p(c)$ for equilibrium prices $p(c)$ and some μ. Inverse demand $D(q(c))$ for a monopolist with cost c is therefore $D(q(c)) = \frac{u'(q(c))}{\mu}$. In equilibrium a monopolist with costs c picks $q_m(c)$ according to

$$\max_{q_m(c)}[D(q_m(c)) - c]q_m(c) = \max_{q_m(c)}[\frac{u'(q_m(c))}{\mu} - c]q_m(c) \tag{MP}$$

so long as the resulting profit covers f. By assumption, the FOC $[\frac{u'(q_m(c))}{\mu} - c] + \frac{u''(q_m(c))}{\mu}q_m(c) = 0$ uniquely determines each monopolist's optimal quantity which must be $q^*(c)$ in equilibrium. We conclude that $q^*(c)$ is implicitly determined by the monopolist FOC as given in Equation (7).

$$u'(q^*(c)) + u''(q^*(c))q^*(c) = \mu c \tag{7}$$

We now show $q^* = q_p$. Since $H(q_p) = H(q^*)$ and $H(q)$ is linear in q, any convex combination $q_\alpha \equiv \alpha q^* + (1-\alpha)q_p$ has $H(q_\alpha) = H(q_p) = H(q^*)$ and so is attainable. Since u is strictly concave, a standard concavity argument shows that the optimality of q_p and q^* implies $q_p = q_\alpha = q^* \quad \forall \alpha \in [0,1]$. Now comparing Equations (6) and (7) with the knowledge that $q^* = q_p$ and dividing the second by the first we see Equation (8) holds on $[0, c_d]$.

$$1 + \frac{u''(q_p(c))q_p(c)}{u'(q_p(c))} = \frac{\mu}{\lambda'} \tag{8}$$

Equation (8) implies for some constant k_0 that for each $c \in [0, c_d]$ that

$$u''(q_p(c))q_p(c) \quad = \quad k_0 u'(q_p(c))$$

Equation (7) paired with $u'' < 0$ shows that $q(c)$ is strictly decreasing so we have that $q([0, c_d]) = [q(c_d), q(0)]$. Consequently, $\forall x \in [q(c_d), q(0)]$ we have that $u''(x)x = k_0 u'(x)$. Standard solution techniques imply that the unique continuously differentiable solution for u on $[0, c_d]$ is $u(x) = \alpha + \beta x^\gamma$ for constants α, β, γ, which is precisely the CES form up to an affine transformation.

A.3 Independence of Selection and Scaling

Proposition. *Under preferences as in Equation (5), economy size does not matter for the cost cutoff level, prices and level of varieties.*

Proof. We will show that for any two Melitz economies of size L and L', the market equilibrium outcomes are the same excepting M and M_e. First consider a market equilibrium of an economy of size L which has characterizing conditions (1), (2) and (3):

1. Consumers maximize utility given prices and income. Thus $u'(q(c)) = \lambda p(c)$ where λ is a positive constant which solves $\frac{L}{M_e} = \int_0^{c_D} p(c)(u')^{-1}(\lambda p(c))dG$ and $q(c) = (u')^{-1}(\lambda p(c))$.

2. Subsequent to entry, firms maximize profits which implies for $h(x) \equiv (u')^{-1}(x)$, $h(\lambda p(c)) + \lambda(p(c) - c)h'(\lambda p(c)) = 0$.

3. Prior to entry, expected industry profits are zero. $\int_0^{c_D}(p(c) - c)h(\lambda p(c))dG = fG(c_D) + f_e$.

In order to construct the equilibrium for an economy of size L', let denote $'$ denote the equilibrium values under L' and begin with let $M'_e \equiv \frac{L'}{L}M_e$ and $p'(c) \equiv p(c)$. Then $\frac{L'}{M'_e} = \frac{L}{M_e} = \int_0^{c_D} p(c)h(\lambda p(c))dG$ so from Condition (1), $\lambda' = \lambda$ and $q'(c) = q(c)$. This implies at $\lambda', q'(c)$ both Conditions (2) and (3) hold so $\{M'_e = \frac{L'}{L}M_e,\ p'(c) = p(c),\ q'(c) = q(c)\}$ constitute and equilibrium for an economy of size L'.

A.4 Melitz Open Economy

Proposition. *Assuming $(f/f_x)^{\frac{1-\rho}{\rho}} < \tau$ (as in Melitz). Then the market equilibrium of an open Melitz economy is socially optimal.*

Proof. Fix a country i and that country's contribution to aggregate welfare is given by W^i where

$$W^i(\tilde{q}^i, \tilde{M}^i_e, \tilde{c}^i_d) \equiv M^i_e \int_0^{c^i_d} q^i(c)^\rho g(c)dc + \sum_{j \neq i} \tau^{-\rho} M^j_e \int_0^{c^j_d} q^j(c)^\rho g(c)dc$$

Since labor is not mobile, maximizing $W \equiv \sum W^j(\tilde{q}^j, \tilde{M}^j_e, \tilde{c}^j_d)$ is equivalent to maximizing each W^j separately. In particular, if we can exhibit a maximum for W^i which corresponds to the market equilibrium then by symmetry we are done. Defining $U^j(q^j, M^j_e, c^j_d) \equiv M^j_e \int_0^{c^j_d} q^j(c)^\rho g(c)dc$, we have $W^i = U^i + \tau^{-\rho}\sum_{j\neq i} U^j$.[22] Explicitly, suppressing some arguments the problem of maximizing W^i is given by Equation (9) where H^i is given by Equation (10).

$$\max_{\tilde{q}^i, \tilde{M}^i_e, \tilde{c}^i_d} U^i + \tau^{-\rho}\sum_{j\neq i} U^j \text{ subject to } H^i(\tilde{q}^i, \tilde{M}^i_e, \tilde{c}^i_d) = 0 \tag{9}$$

$$H^i(\tilde{q}^i, \tilde{M}^i_e, \tilde{c}^i_d) \equiv L - \sum_j M^j_e \int_0^{c^j_d} cq^j(c)g(c)dc - \max_j\{M^j_e\}[f_e + G(\max_j\{c^j_d\})f] - f_x \sum_{j\neq i} G(c^j_d)M^j_e \tag{10}$$

Again suppressing the arguments $\tilde{q}^i, \tilde{M}^i_e, \tilde{c}^i_d$ and for brevity defining the two "max" terms as

$$\overline{M} \equiv \max_j\{M^j_e\} \qquad \overline{c} \equiv \max_j\{c^j_d\}$$

we can decompose the allocation of labor by export destination and a fixed cost component L_f as $H^i = L - [L_f + \sum L_j]$ where

$$L_f(\overline{M}, \overline{c}) \equiv \overline{M}[f_e + G(\overline{c})f] \;\; L_i \equiv M^i_e \int_0^{c^i_d} cq^i(c)g(c)dc \;\; L_j \equiv M^j_e[\int_0^{c^j_d} cq^j(c)g(c)dc + f_x G(c^j_d)]$$

[22] Our strategy is to reduce optimal welfare W^i to a function of $\overline{M}$ and the L_j. This reduces Equation (9) to a simpler maximization problem whose solution is that of the market equilibrium.

For the present fix $\overline{M}$ and the L_j and consider the problem of maximizing each U^j individually whenever $L_j > 0$. Maximizing each U^j for $j \neq i$ are equivalent problems, with maximizing U^i being slightly different since fixed costs f_x are not incurred. The two maximization problems for U^j and U^i are given respectively by Equations (11) and (11) with the additional constraints $M_e^j, M_e^i \leq \overline{M}$ and $c_d^j, c_d^i \leq \overline{c}$.

$$\max_{q^j, M_e^j, c_d^j} U^j(q^j, M_e^j, c_d^j) \text{ subject to } L_j = M_e^j\left[\int_0^{c_d^j} cq^j(c)g(c)dc + f_x G(c_d^j)\right] \tag{11}$$

$$\max_{q^i, M_e^i, c_d^i} U^i(q^i, M_e^i, c_d^i) \text{ subject to } L_i = M_e^i \int_0^{c_d^i} cq^i(c)g(c)dc \tag{12}$$

Following the same strategy to solve Equations (11) and (12) as in the autarky case, by fixing $M_e^j, M_e^i \leq \overline{M}$ and $c_d^j, c_d^i \leq \overline{c}$ we have that the unique solution for $q^j(c)$ and $q^j(c)$ which are sufficient for an optimum for M_e^j, M_e^i and c_d^j, c_d^i fixed (provided $L_j > M_e^j f_x G(c_d^j)$ which must hold as above so long as $L_j > 0$) are given by Equation (13) (where as above $R(x) \equiv \int_0^x c^{\frac{\rho}{\rho-1}} g(c)dc$).

$$q^j(c) = c^{\frac{1}{\rho-1}}\left[\frac{L_j}{M_e^j} - f_x G(c_d^j)\right]/R(c_d^j) \qquad q^i(c) = c^{\frac{1}{\rho-1}}\left[\frac{L_i}{M_e^i}\right]/R(c_d^j) \tag{13}$$

Equation (13) respectively gives welfare U^j and U^i of

$$U^j = (M_e^j)^{1-\rho}(L_j - M_e^j f_x G(c_d^j))^\rho R(c_d^j)^{1-\rho} \qquad U^i = (M_e^i)^{1-\rho} L_i^\rho R(c_d^i)^{1-\rho} \tag{14}$$

so long as $L_j > 0 \quad \forall j$. In fact, since $\rho < 1$, so long as $L_j > 0$ for at least one $j \neq i$, it is easy to show that a necessary condition is that $L_j > 0 \quad \forall j \neq i$ by reallocating labor from the positive L_j to some $L_k = 0$ so either all $L_{j \neq i}$ are strictly positive or zero. Clearly $f_x > 0$ also implies a necessary condition is that $L_i \geq \max_{j \neq i}\{L_j\}$ so the only cases we need to consider are $L_j > 0 \quad \forall j$ and $L_j = 0 \quad \forall j \neq i$.

We now summarize what we have shown so far. We have reduced the problem (9) to two finite dimensional problems (15-16) given below corresponding to the cases above regarding the L_j, respectively.[23]

$$\max_{\{L_k\}, \tilde{M}_e^i, \tilde{c}_d^i, \overline{M}, \overline{c}} (M_e^i)^{1-\rho} L_i^\rho R(c_d^i)^{1-\rho} + \tau^{-\rho} \sum_{j \neq i} (M_e^j)^{1-\rho}(L_j - M_e^j f_x G(c_d^j))^\rho R(c_d^j)^{1-\rho} \tag{15}$$

$$\text{subject to } L_f(\overline{M}, \overline{c}) + \sum L_j = L \text{ and } M_e^j \leq \overline{M}, \quad c_d^j \leq \overline{c} \quad \forall j$$

$$\max_{L_i, \overline{M}, \overline{c}} (\overline{M})^{1-\rho} L_i^\rho R(\overline{c})^{1-\rho} \qquad \text{subject to } L_f(\overline{M}, \overline{c}) + L_i = L \tag{16}$$

Now for either problem (15-16) and any fixed pair $(\overline{M}, \overline{c})$ the remaining choice variables are restricted to a compact set $K(\overline{M}, \overline{c})$ so that continuity of the objective function (by defining $U_j = 0$ when $L_j = 0$) guarantees a solution to the problem exists for each pair $(\overline{M}, \overline{c})$ and we denote the value of the objective function at the maximum by $S(\overline{M}, \overline{c})$. In fact, $K(\overline{M}, \overline{c})$ can be shown

[23] For Problem (16) it is clear that $M_e^{i*} = \overline{M}$ and $c_d^{i*} = \overline{c}$.

to be a continuous correspondence so by the Theorem of the Maximum $S(\overline{M},\overline{c})$ is continuous on $L_f^{-1}([0,L])$ which is compact since L_f is continuous and therefore a global max of $S(\overline{M},\overline{c})$ exists which is the Social Optimum by the above arguments (see for instance, Berge and Karreman (1963)).

Problem (16) corresponds exactly to a Planner in autarky and we have detailed the solution above, namely an allocation corresponding to the market equilibrium in autarky. As shown in Melitz, under our parameter assumption the open economy market equilibrium (clearly attainable by a social planner) yields higher welfare than the closed market equilibrium and therefore the solution to Problem (15) yields higher welfare than Problem (16).

As for Problem (15), having shown existence, we now return to maximizing Equations (14) for fixed L_i, L_j. The solution for the $j = i$ problem is clearly $M_e^i = \overline{M}$, $c_d^i = \overline{c}$. Consider any j where $L_j > 0$ so clearly we must have $M_e^j, c_d^j > 0$ at any optimum. One may show using standard techniques that for fixed c_d^j, U^j is strictly concave in M_e^j with critical point $\frac{(1-\rho)L_j}{f_x G(c_d^j)}$ so for any fixed c_d^j, $M_e^{j*} = \min\{\frac{(1-\rho)L_j}{f_x G(c_d^j)}, \overline{M}\}$. Now if the optimal c_d^j is s.t. $c_d^j \in (0,\overline{c})$ we must have for fixed M_e^{j*} that $\frac{\partial U^j}{\partial c_d^j} = 0$. Some algebra shows that this FOC is equivalent to Equation (17).

$$[L_j - M_e^{j*} f_x G(c_d^j)] = \frac{\rho}{1-\rho} \frac{M_e^{j*} f_x R(c_d^j)}{(c_d^j)^{\frac{\rho}{\rho-1}}} \tag{17}$$

Similarly if for fixed c_d^j we have $M_e^{j*} = \frac{(1-\rho)L_j}{f_x G(c_d^j)} < \overline{M}$ we must have

$$[L_j - M_e^{j*} f_x G(c_d^j)] = \frac{\rho}{1-\rho} M_e^{j*} f_x G(c_d^j) \tag{18}$$

Equations (17) and (18) together would imply $\frac{R(c_d^j)}{(c_d^j)^{\frac{\rho}{\rho-1}} G(c_d^j)} = 1$ but $\frac{R(c_d^j)}{(c_d^j)^{\frac{\rho}{\rho-1}} G(c_d^j)} > 1$ for $c_d^j > 0$ so we conclude that $c_d^j < \overline{c}$ implies $M_e^{j*} = \overline{M} < \frac{(1-\rho)L_j}{f_x G(c_d^j)}$ and $\frac{(1-\rho)L_j}{f_x G(c_d^j)} < \overline{M}$ implies $c_d^j = \overline{c}$. Now considering $M_e^{j*} = \frac{(1-\rho)L_j}{f_x G(c_d^j)} < \overline{M}$ as a function of c_d^j, from above we have

$$\text{sgn}\{\frac{\partial U^j(M_e^{j*}, c_d^j)}{\partial c_d^j}\} = \text{sgn}\{(1-\rho)\frac{(c_d^j)^{\frac{\rho}{\rho-1}}}{R(c_d^j)} - \rho \frac{M_e^{j*} f_x}{L_j - M_e^{j*} f_x G(c_d^j)}\} = \text{sgn}\{(1-\rho)\frac{(c_d^j)^{\frac{\rho}{\rho-1}} G(c_d^j)}{R(c_d^j)} - (1-\rho)\} < 0$$

since again $\frac{R(c_d^j)}{(c_d^j)^{\frac{\rho}{\rho-1}} G(c_d^j)} > 1$ for $c_d^j > 0$. This implies that for $M_e^{j*} = \frac{(1-\rho)L_j}{f_x G(c_d^j)} < \overline{M}$, $c_d^j = \overline{c}$ is not an optimum and therefore at any optimum, $M_e^{j*} = \overline{M} \leq \frac{(1-\rho)L_j}{f_x G(c_d^j)}$. Since $c_d^{j*} = 0$ is clearly not optimal, this in turn implies that either Equation (17) holds or $c_d^{j*} = \overline{c}$. With these reductions we revisit Problem 15 which has been reduced to Problem 19 with the added constraints of either $c_d^j = \overline{c}$ or

Equation (17) holds $\forall j \neq i$.

$$\max_{\{L_k\},\bar{c}_d^i,\overline{M},\bar{c}} \overline{M}^{1-\rho}\{L_i^{\rho}R(\bar{c})^{1-\rho} + \tau^{-\rho}\sum_{j\neq i}(L_j - \overline{M}f_x G(c_d^j))^{\rho}R(c_d^j)^{1-\rho}\} \text{ sub to } L_f(\overline{M},\bar{c}) + \sum L_j = L \tag{19}$$

Now consider solving Problem (19) with c_d^j unconstrained. Using a standard Lagrangian approach, we find a candidate solution from necessary conditions in which $c_d^{j*} = (\frac{f_x}{f})^{\frac{\rho-1}{\rho}}\frac{\bar{c}}{\tau}$ and since it is assumed $(\frac{f}{f_x})^{\frac{1-\rho}{\rho}} < \tau$, $c_d^{j*} < \bar{c}$. The candidate solution with c_d^j unconstrained also yields Equation (17) so the unconstrained candidate solution satisfies the omitted constraints. We conclude the necessary conditions embodied in the candidate solution are also necessary for any solution to Problem (19). It turns out these necessary conditions are exactly those which fix market equilibria so if the market equilibrium exists and is unique, there is a unique solution to the necessary conditions and since existence of a solution to Problem (19) was shown above, the planner and market allocations coincide.

A.5 Converse of Optimality of Selection Effects with CES-Benassy

Proposition 6. *Consider a Melitz economy with preferences as in (1) and Benassy ν. The market equilibrium is socially optimal only if u is CES and $\nu_B = (1-\rho)/\rho$ as in Melitz (2003).*

Proof. Under Benassy preferences, specifically $U(M_e, c_d, q)$ where

$$U(M_e, c_d, q) \equiv \{[M_e G(c_d)]^{\rho(\nu+1)-1} M_e \int_0^{c_d} q(c)^{\rho} g(c)dc\}^{\rho}$$

one may show that the market equilibrium M_e, c_d and $q(c)$ correspond to the market equilibrium of Melitz (2003). One may also show that for fixed M_e and c_d a social planner will choose q^* under Benassy as under Melitz as in the proof that a Melitz economy is first best. Having fixed q^* as in that proof, we consider the reduced SP problem which involves finding an optimal M_e and c_d.

Finding M_e for c_d fixed. We may therefore consider maximizing $W(M_e, c_d)$ where we have

$$W(M_e, c_d) \equiv [M_e G(c_d)]^{\rho(\nu+1)-1} M_e \int_0^{c_d} q^*(c)^{\rho} g(c)dc = [M_e G(c_d)]^{\rho(\nu+1)-1} M_e [L/M_e - f_e - fG(c_d)]^{\rho} R(c_d)^{1-\rho}$$

and for c_d fixed we can maximize $\tilde{W}(M_e) \equiv M_e^{\nu}[\gamma - M_e]$ where $\gamma \equiv L/(f_e + fG(c_d))$ since the arg max of $\tilde{W}(M_e)$ maximizes $W(M_e, c_d)$ for c_d fixed. Looking at the FOC we have

$$\tilde{W}'(M_e) \quad = \quad M_e^{\nu-1}\{\nu\gamma - (\nu+1)M_e\}$$

so the unique maximum of $\tilde{W}$ is

$$M_e^*(c_d) = \frac{\nu}{\nu+1} L/(f_e + fG(c_d))$$

Inefficiency of the market equilibrium. Under Melitz preferences we have shown the SP chooses c_d to maximize $Z(c_d)$ where

$$\begin{aligned} Z(c_d) &\equiv \rho^{\rho}(1-\rho)^{1-\rho} L R(c_d)^{1-\rho}[f_e + fG(c_d)]^{\rho-1} \\ &\propto R(c_d)^{1-\rho}[f_e + fG(c_d)]^{\rho-1} \end{aligned}$$

and the arg max of Z corresponds to the c_d fixed by the market, say c_d^M. Now we can compute an explicit measure of welfare using $M_e^*(c_d)$ which is

$$W(M_e^*(c_d), c_d) = [\frac{G(c_d)}{f_e + fG(c_d)}]^{\rho(\nu+1)-1} L^{\rho(\nu+1)} [\frac{\nu}{\nu+1}]^{\rho\nu} [\frac{1}{\nu+1}]^{\rho} R(c_d)^{1-\rho} (f_e + fG(c_d))^{\rho-1}$$

$$\propto \quad [\frac{G(c_d)}{f_e + fG(c_d)}]^{\rho(\nu+1)-1} Z(c_d)$$

Since $W(M_e^*(c_d), c_d)$ and $Z(c_d)$ are continuously differentiable in c_d, by the above remarks we must have $Z'(c_d^{ME}) = 0$ and if the market is efficient under Benassy that $\frac{\partial}{\partial c_d} W(M_e^*(c_d^M), c_d^M) = 0$. But then

$$\begin{aligned}
\frac{\partial}{\partial c_d} W(M_e^*(c_d), c_d) &\propto \frac{\partial}{\partial c_d} \{ [\frac{G(c_d)}{f_e + fG(c_d)}]^{\rho(\nu+1)-1} Z(c_d) \} \\
&= \{ \frac{\partial}{\partial c_d} [\frac{G(c_d^M)}{f_e + fG(c_d^M)}]^{\rho(\nu+1)-1} \} Z(c_d^M) + [\frac{G(c_d^M)}{f_e + fG(c_d^M)}]^{\rho(\nu+1)-1} Z'(c_d^M) \\
&= (\rho(\nu+1) - 1) [\frac{G(c_d^M)}{f_e + fG(c_d^M)}]^{\rho(\nu+1)-2} \frac{g(c_d^M) f_e}{[f_e + fG(c_d^M)]^2} Z(c_d^M)
\end{aligned}$$

This last expression is zero iff $\nu = \frac{1-\rho}{\rho}$ so we conclude that even if preferences are CES, the market is efficient under Benassy preferences iff $\nu = \frac{1-\rho}{\rho}$.

A.6 Market Equilibrium in Melitz and MO

The equilibrium outcomes are summarized in Tables 3 and 4. We denote the domestic autarky outcomes by subscript a and the open economy outcomes by t. The domestic and export markets of the open economy are denoted by d and x respectively. For brevity, we define $\tilde{c}_i^{\rho/(\rho-1)} \equiv \int_0^{c_i} c^{\rho/(\rho-1)} dG/G(c_i)$ and $k(c_i) \equiv (\tilde{c}_i/c_i)^{\rho/(\rho-1)} - 1$ for $i = a, d, x$. Note that $f = 0$ in MO.

Table 3: Market Outcomes in Autarky (a)

	Melitz (2003)	MO (2008)
c_a	$f_e/G(c_a) = fk(c_a)$	$c_a^{k+2} = 2(k+1)(k+2)\gamma c_M^k f_e/L$
$p(c)$	c/ρ	$[c_a + c]/2$
$\tilde{p}_a$	$\tilde{c}_a/\rho$	$[c_a + \tilde{c}(c_a)]/2 = (2k+1)c_a/2(k+1)$
M_a	$\frac{(1-\rho)L}{f_e/G(c_a)+f}$	$\frac{2(k+1)\gamma}{\eta} \frac{\alpha - c_a}{c_a}$

Table 4: Market Outcomes in the Open Economy (t)

	Melitz (2003)	MO (2008)
c_d	$f_e/G(c_d) = fk(c_d) + nG(c_x)f_x k(c_x)/G(c_d)$	$c_d^{k+2} = \frac{1-(\tau^*)^{-k}}{1-(\tau^*\tau)^{-k}}[2(k+1)(k+2)\gamma c_M^k f_e/L]$
c_x	$\frac{c_d}{\tau}\left(\frac{f}{f_x}\right)^{\frac{1-\rho}{\rho}}$	$\frac{c_d^*}{\tau^*}$
$\tilde{c}_t(c_d)$	$\left\{\frac{M_d}{M_t}\tilde{c}_d^{\frac{\rho}{\rho-1}} + n\frac{M_x}{M_t}(\tau\tilde{c}_x)^{\frac{\rho}{\rho-1}}\right\}^{\frac{\rho-1}{\rho}}$	$\frac{k}{k+1}c_d$
$p_x(c)$	$\tau c/\rho$	$[c_x + \tau^* c]/2$
$\tilde{p}_t$	$\tilde{c}_t/\rho$	$(2k+1)c_d/2(k+1)$
M_t	$\frac{(G(c_d)+nG(c_x))(1-\rho)L}{f_e+fG(c_d)+nf_xG(c_x)}$	$\frac{2(k+1)\gamma}{\eta}\frac{\alpha-c_d}{c_d}$

d and x denote the domestic and export markets in the open economy respectively.